**1.** A curve has equation $7x^2 + 48xy - 7y^2 + 75 = 0$.

*A* and *B* are two distinct points on the curve. At each of these points the gradient of the curve is equal to $\frac{2}{11}$.

(a) Use implicit differentiation to show that $x + 2y = 0$ at the points *A* and *B*.

**(5)**

(b) Find the coordinates of the points *A* and *B*.

**(4)**
**(Total 9 marks)**

**2.** Differentiate with respect to $x$

(i) $x^3 e^{3x}$,

**(3)**

(ii) $\dfrac{2x}{\cos x}$,

**(3)**

(iii) $\tan^2 x$.

**(2)**

Given that $x = \cos y^2$,

(iv) find $\dfrac{dy}{dx}$ in terms of $y$.

**(4)**
**(Total 12 marks)**

| | | | | |
|---|---|---|---|---|
| **1.** | (a) | $14x + (48x\frac{dy}{dx} + 48y) - 14y\frac{dy}{dx} = 0$ | M1 (B1) A1 | |
| | | Substitutes $\frac{dy}{dx} = \frac{2}{11}$ into derived expression to obtain | M1 | |
| | | $14x + \frac{96}{11}x + 48y - \frac{28}{11}y = 0$ | | |
| | | $\therefore 250x + 500y = 0 \quad \Rightarrow \quad x + 2y = 0$ | A1 | 5 |
| | (b) | Eliminates one variable to obtain, for example, $7(2y)^2 + 48(-2y)y - 7y^2 + 75 = 0$ and obtains $y$ (or $x$) | | |
| | | Substitutes $y$ to obtain $x$ (or $y$) | M1 | |
| | | Obtains coordinates (–2,1) and (2,–1) | A1, A1 | 4 |

**[9]**

| | | | | |
|---|---|---|---|---|
| **2.** | (i) | $u = x^3 \quad \frac{du}{dx} = 3x^2$ | | |
| | | $v = e^{3x} \quad \frac{dv}{dx} = 3e^{3x}$ | | |
| | | $\frac{dy}{dx} = 3x^2 e^{3x} + x^3 3e^{3x}$ or equiv | M1 A1 A1 | 3 |
| | (ii) | $u = 2x \quad \frac{du}{dx} = 2$ | | |
| | | $v = \cos x \quad \frac{dv}{dx} = -\sin x$ | | |
| | | $\frac{dy}{dx} = \frac{2\cos x + 2x\sin x}{\cos^2 x}$ or equiv | M1 A1 A1 | 3 |
| | (iii) | $u = \tan x \quad \frac{du}{dx} = \sec^2 x$ | | |
| | | $y = u^2 \quad \frac{dy}{du} = 2u$ | | |
| | | $\frac{dy}{dx} = 2u \sec^2 x$ | M1 | |
| | | $\frac{dy}{dx} = 2\tan x \sec^2 x$ | A1 | 2 |

(iv) $u = y^2$ $\qquad \dfrac{\mathrm{d}u}{\mathrm{d}y} = 2y$

$x = \cos u$ $\qquad \dfrac{\mathrm{d}x}{\mathrm{d}u} = -\sin u$ M1

$\dfrac{\mathrm{d}x}{\mathrm{d}y} = -2y \sin y^2$ A1

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-1}{2y \sin y^2}$ M1 A1 4

**[12]**

**1.** In part (a) there was a lot of good work for the implicit differentiation, with many candidates scoring the first three marks. For a significant group of candidates however, the $48xy$ was a problem – they did not treat it as a product. A number had $48\frac{dy}{dx}$ for this. Although many candidates had an extra $\frac{dy}{dx}$ term at the start, this usually disappeared when they started to rearrange their equation to make $\frac{dy}{dx}$ the subject of an equation. Not all candidates could see how to proceed once they had substituted the $\frac{2}{11}$. Too many candidates had an expression with two fractions equal and simply equated the numerators and equated the denominators, showing some lack of understanding of equal fractions.

Part (b) proved more difficult than expected. Many candidates indicated an attempt to substitute to obtain an equation in just one variable, but the work was often full of arithmetic and algebraic errors. Too many used long methods to find the second variable – preferring to go back to the quadratic equation rather than use $x = -2y$. Others began again, with two equations, and eliminated the second variable. Correct final answers were relatively rare and a large number of candidates wrote their co-ordinates as $(y, x)$ not $(x, y)$. This was disappointing at this level.

**2.** No Report available for this question.